Time Dilation and Length Contraction Formulas.

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Introduction

This worksheet demonstrate the simplicity of deriving the Time Dilation and Length Contraction Formulas using a basic mathematical theory (Pythagorean theorem) Step by Step which follow from the Principle of Relativity. It's simple in all directions. Events play a major role in Relativity because they involve space and time. So for every event to occur there is a place(Space) and time, the two events are not the same because they involve different time and place(Space). So in order to visualize the behavior of this phenomena rather than using the traditional "Two Observers" example, I will use a "Bouncing Light Beam situation" which i adopted from professor RICHARD WOLFSON book "Simply Einstein, Relativity Demystified". Rather than describing only light and how it bounce with respect to different reference frame but it's about the nature of Time.

Initialization

```
> restart:
> with(ScientificConstants):
> with(geometry):
```

Pythagorean Theorem Revision

Time Dilation Equation depends on the Pythagorean Theorem which gives the hypotenuse of a right triangle in terms
of the lengths of the other two sides. So, we know that to get the length of the hypotenuse we need to squared the
other two sides and "radicalize" them.

Maple Visualization

Define triangle M with it's coordinates (Target to be Right Triangle), and labeling each side.

```
> triangle(M,[point(A1,0,0),point(B1,0,5),point(C1,8,0)]);
```

$$M \quad (3.1.1)$$

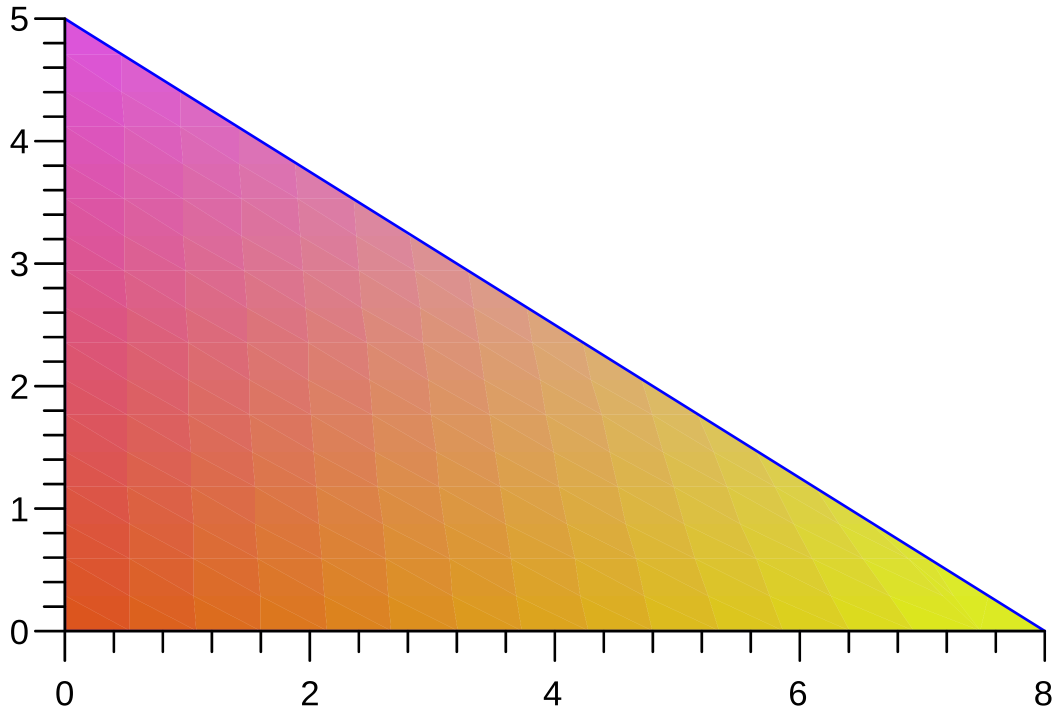
Check if it's Right Triangle

```
> IsRightTriangle(M);
```

$$true \quad (3.1.2)$$

Draw the Right Triangle

```
> draw([M(color='COLOR'(RGB,1.00000000,1.00000000,.8000000000),
  filled=true),
  M(color=blue)]);
```



In order to get the length of the hypotenuse, If we labeled the X-axis "A1" and Y-axis "B1" therefore "C1" equal:

```
> C1 = sqrt((A1^2)+(B1^2));
```

$$C1 = \sqrt{A1^2 + B1^2} \tag{3.1.3}$$

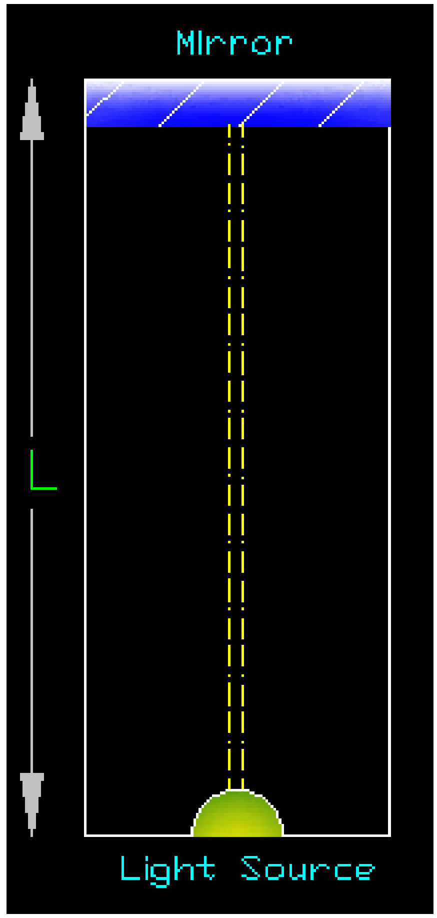
Time Dilation and Contraction Formulas

Box at rest

We know that

```
> Distance = Speed * Time;
```

$$Distance = Speed\ Time \tag{4.1.1}$$



Light path denoted by dashed yellow line, and L denote the Length.
In This Figure (Box is shown in a reference frame where it's at rest) You Will Notice That Light Goes A Round-Trip 2L Between Source, Bouncing Off The Mirror , And Returning To The Source .

If We Apply Distance = Speed * Time Equation To Obtain The Time (Speed Of Light) We Get:′

```
> R1 := (2 * L = C * Tprime);
```

$$R1 := 2\,L = \left(speed\_of\_light\_in\_vacuum,\ symbol = c,\ value = 299792458,\ uncertainty = 0,\ units = \frac{m}{s}\right) Tprime \tag{4.1.2}$$

Where (the time in this reference frame called t' (Tprime), GetConstant (c) = Speed Of Light).
Dividing both sides of R1 by C:

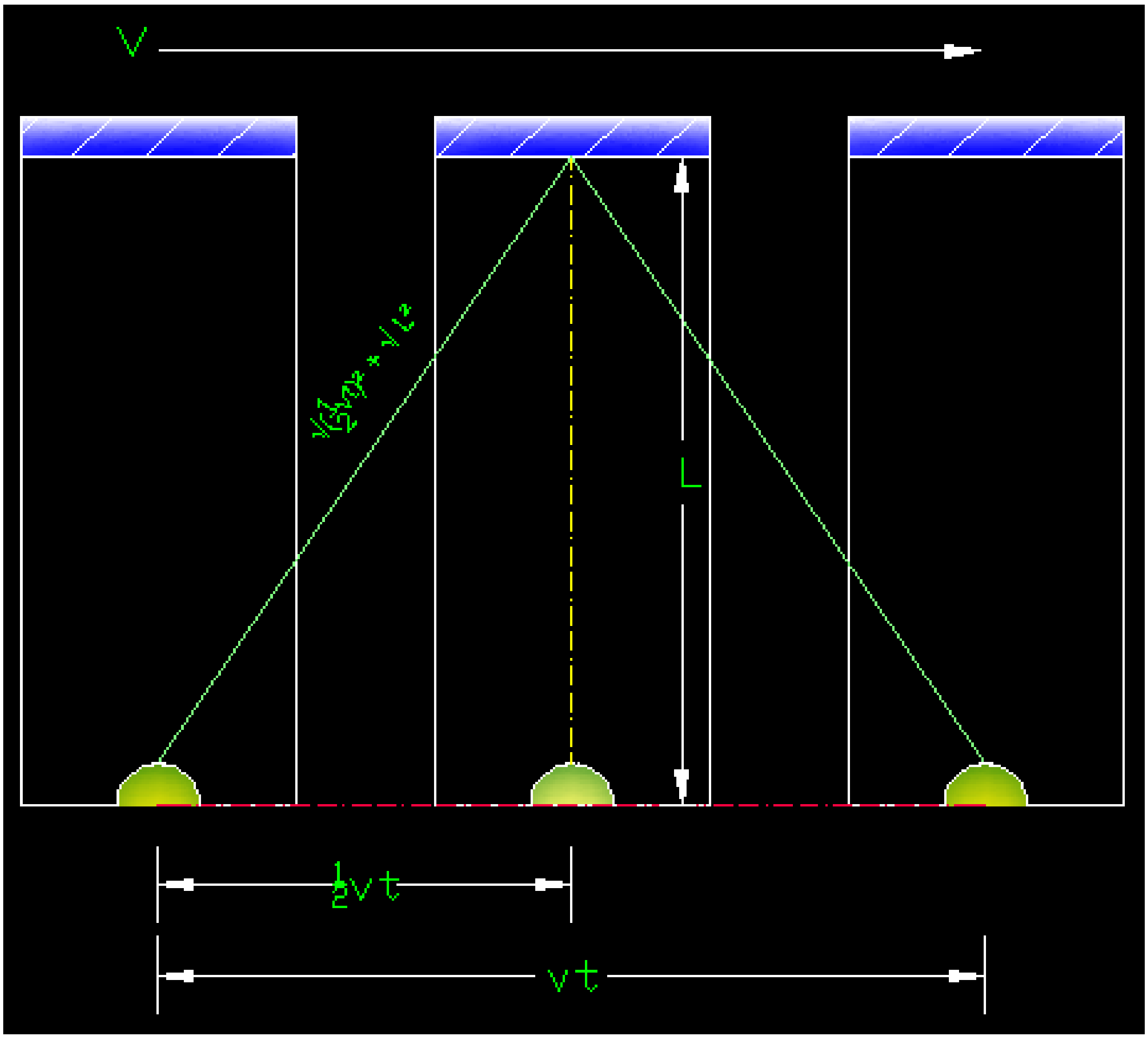
```
> R2 := Tprime = 2*L / C;
```

$$R2 := Tprime = (2\,L) \Big/ \left(speed\_of\_light\_in\_vacuum,\ symbol = c,\ value = 299792458,\ uncertainty = 0,\ units = \frac{m}{s}\right) \tag{4.1.3}$$

```
>
```

Box is moving

"Now let's go illustrate the frame in which the box is moving at speed v, and we're calling t the time between departure and return of the light flash.. So distance = Speed * Time tells us that the distance the box travels is just vt. While the box goes half this distance, the light itself takes the diagonal path up to the mirror, where the long of this diagonal is the hypotenuse of a right triangle with sides $\frac{1\,v\,t}{2}$; *and L*." Professor RICHARD WOLFSON



The pythagorean theorem then gives it's length:

```
> x1 := sqrt(((1/2*v*T)^2)+(L^2));
```

(4.2.1)

$$x1 := \frac{1}{2}\sqrt{v^2 T^2 + 4 L^2} \qquad (4.2.1)$$

The total light path is twice this distance, the time the light takes is T, speed of light is C therefore:

```
> x2 := (2 * x1 = C*T);
```

$$x2 := \sqrt{v^2 T^2 + 4 L^2} = C T \qquad (4.2.2)$$

Now we want to know the timt T, because it's inside the square root, we need to square both sides:

```
> x3 := (2 * x1)^2 = (C * T)^2;
```

$$x3 := v^2 T^2 + 4 L^2 = C^2 T^2 \qquad (4.2.3)$$

And now by subtracting $\left(v^2 T^2\right)$; from both sides we get:

```
> x4 := (x3-((v^2)*(T^2)));
```

$$x4 := 4 L^2 = C^2 T^2 - v^2 T^2 \qquad (4.2.4)$$

Final step: divide both sides by c^2; , and then squared both side:

```
> x5 := (x4)/(C^2);
```

$$c^2$$

$$x5 := \frac{4 L^2}{C^2} = \frac{C^2 T^2 - v^2 T^2}{C^2} \qquad (4.2.5)$$

```
>
```

Partitionized the x5 statement into two section: x6 := $\frac{4 L^2}{C^2}$ and x7 := $\frac{C^2 T^2 - v^2 T^2}{C^2}$

```
> x6 := sqrt(4*L^2/C^2);
```

$$x6 := 2\sqrt{\frac{L^2}{C^2}} \qquad (4.2.6)$$

```
> x7 := sqrt((C^2*T^2-v^2*T^2)/C^2);
```

$$x7 := \sqrt{\frac{C^2 T^2 - v^2 T^2}{C^2}} \qquad (4.2.7)$$

```
> x8 := simplify(x6 = x7);
```

$$x8 := 2\sqrt{\frac{L^2}{C^2}} = \sqrt{-\frac{T^2\left(-C^2 + v^2\right)}{C^2}} \qquad (4.2.8)$$

```
> x9 := ((2*L/C) = (T*sqrt(1-(v^2)/(C^2))));
```

$$x9 := \frac{2 L}{C} = T\sqrt{1 - \frac{v^2}{C^2}} \qquad (4.2.9)$$

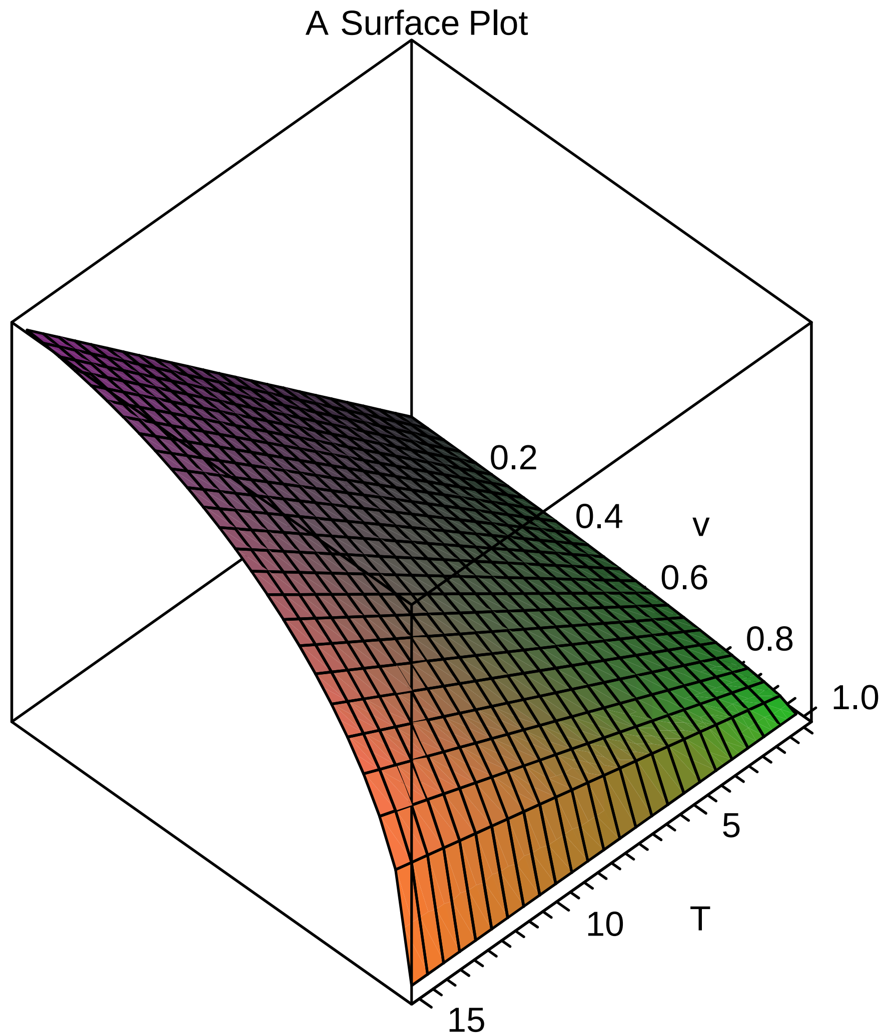
And we have $\frac{2 L}{C} = Tprime$ in the Box at rest section ∴

```
> x10 := ((Tprime) = (T*sqrt(1-(v^2)/(C^2))));
```

(4.2.10)

$$x10 := Tprime = T\sqrt{1 - \frac{v^2}{C^2}} \quad (4.2.10)$$

```
> plot3d(T * sqrt(1-v^2) , T=1..15, v=0.1..1, axes=BOXED,
  lightmodel=light1,title="A Surface Plot" );
```



References

WOLFSON, RICHARD. SIMPLY EINSTEIN, Relativity Demystified. United States. New Yorkl: W. W. Norton & Company, Inc., 2003.

Conclusion

We've seen through this small journey a zipped revision about Pythagorean theorem, and then we derived the time dilation equation and length contraction formula based on this simple theorem and we demonstrate that it's indeed follow directly from the Special Theory of Relativity.

I hope that you understand it well; and to go deep in the analysis of the construction of the last formula. $x10 := Tprime = T\sqrt{1-\frac{v^2}{C^2}}$, i advise you to prove to yourself why every variable in this formula arranged like this and why there is "One" under the square root as a result.Imagine yourself rearrenging
every variable in this formula mentally and by hand on the paper.

Friday, December 9, 2005

Disclaimer