

On the Intractability of Designing an Efficient Entropy Brute Forcer

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Abstract

The problem of finding a contiguous set of bytes that have a given entropy value in a binary is a multifaceted undertaking. This is due to unpredictable and hard to 'patternize' formations of various bytes distribution in a given search space. This is an inherent limitation in the way entropy works, since without prior knowledge about the distribution of bytes, the expectancy of determining any subsequent byte is not guaranteed to converge to any imposed predictor. Thus, scaling (decrease or increase) and extrapolating on multiple spaces does not meet any expected entropy value. In this regard, designing a heuristic approach to tackle this problem is not possible with high accuracy as we will show in this paper. This is important in case where finding arrays of bytes in a binary that have given entropy value helps in determining if its encrypted or packed as discussed in [1]. Moreover, it provides deep inspection capabilities to identify where in the data, blocks of bytes have given entropy value for finding cryptographic elements such as, keys, certificates and others.

We have implemented an entropy brute forcer with five modes of operation, each supporting different accuracy level. More importantly, we provide an optimized brute forcer algorithm which exploits the entropy equation size limit in order to reduce the time complexity when searching for a specific value. In addition, the computational complexity analysis is provided for major operation modes to illustrate the differences in the performance from a mathematical point of view. Furthermore, we used Entyzer (Advanced entropy Analyzer) [2] as the main framework for extending it with the brute force algorithm discussed in this paper. Various Windows binaries have been used in testing the brute forcer.

The results confirm the hypothesis that writing an entropy brute forcer is complex and computationally expensive.

Keywords: Brute Force; Entropy; Entyzer

I. Introduction

“As for me, all I know is that I know nothing”
 - Socrates

In [3], Mokbel and Cambly proposed an unobtrusive entropy based compiler optimization comparator using Shannon entropy as a means to examine the statistical variations at 1-gram byte distribution and quantify the information contained in a binary. In this paper, we build on the extensive analysis presented in [3], especially regarding entropy in depth exploration from theoretical and practical point of views. Thus, interested reader is advised to refer to [3] for more information about entropy.

This paper aims to examine the problem of determining where in a given search space, the sought entropy value is located using flexible parameterized inputs.

For reference, Shannon entropy equation is:

$$H(X) = - \sum_{i=1}^n p(x_i) \log_b p(x_i)$$

The most relevant work to the research presented in this paper is the Entropy IDA Plugin tool developed by Zbitskiy [4]. The tool calculates the entropy for 32-bit PE, ELF and any binary files. In addition, it has the capability to search for a given entropy value based on a chunk and step sizes. However, the tool lacks the powerful input parameterization and the different algorithmic implementations presented in this paper. Thus, the accuracy in locating entropies as well as the performance impact it reveal shows noticeable differences between both works.

II. Algorithmic Analysis

“There are no facts, only interpretations”
- Nietzsche

In this section, we present two algorithms demonstrating various modes of operation. In Algorithm 1, three modes of operation are supported. The algorithm receives three parameterized inputs: *Negative Permissible Range (NPR)*, *Main Value (MV)* and *Positive Permissible Range (PPR)*. The input conditions are stated in \boxed{A} and the conditional operations on those inputs for satisfying given entropy value are located at (L. 13). These values represent the range of the entropy sought target. The output is the address(es) (L. 02) where the sought entropy(ies) H is/are located in the data. The reason behind such a flexible parameterization is to allow for greater possibilities when searching for entropy.

For mode **1** ($m = 1$), from lines [10 – 15], the entropy brute forcer functions by enumerating through all the information \bar{H} , such that when a given entropy value is found, the algorithm starts again from the end address b] of the last found entropy and so on until all the information is consumed. However, this doesn't constitute a true brute forcer.

For mode **2** ($m = 2$), from lines [10 – 17], the entropy brute forcer starts by first exercising mode 1, and if no entropy was found, it keeps advancing the starting offset in the search space by one until it hits the first sought entropy value (if any), and then switches back to mode **1**. Thus, mode **2** is more computationally expensive than mode **1**. This mode registers its worst case and functions as a true brute forcer in case not a single entropy instance was found.

For mode **3** ($m = 3$), from lines [08 – 09], this is the divide and conquer mode. However, it doesn't have a functional implementation on its own. In addition to the previously discussed input parameters, this mode takes only an input d which specifies the number of blocks required to divide the information \bar{H} space. Modes **1**, **2**, **4** and **5** (which will be discussed later in the paper) are all applicable for this mode. This mode represents a fine grained attack on the information space in an attempt to find entropy at the earliest point.

Note that because of the introduced range flexibility when seeking an entropy value, the algorithm is influenced by the earliest entropy match (based on the order of evaluation). Hence, any subsequent matches are subjected to the location of the prior match in the search space. This is due to the function of the indexing as mentioned above.

$$Alg. 1 = \begin{cases} Exit, & H(\bar{H}.Size) = 0.0 & \{1\} \\ \forall(x \in Size_2^{256}) \exists y \in H, & H(\bar{H}.Size) = 8.0 & \{2\} \quad \boxed{B} \\ |H - MV|_{\cong} \leq \varepsilon, & (NPR \wedge PPR) = 0 \wedge (MV \in [0.0, 8.0] \in \mathbb{Q}) & \{3\} \end{cases}$$

Moreover, function \boxed{B} adds more constraints to the algorithm. In case the entropy of the total search space is zero $\boxed{B.1}$, then the algorithm bails out immediately without any further computation. In addition, in case the entropy of the total search space is the maximum entropy value $\boxed{B.2}$, that is 8.0, then a complete holistic heterogeneous search space is detected, which enables finding every possible entropy value in the range between one and eight with respect to every possible length value between 2 and 256. This is illustrated in Table 1 which shows the hexadecimal distribution of a complete 1-gram byte. Furthermore, since

Algorithm 1. ENTROPY BRUTE FORCER

01. **Input:** Information ' \bar{H} ', Operation Mode ' η ', NPR , MV , PPR , d

With the following definitions: $\eta = \{1,2,3\}$ // Modes of operation

$$\boxed{A} \left(\begin{array}{l} (NPR \wedge PPR) \in [0.0, 1.0] \\ MV \in [0.0, 8.0] \end{array} \right) \text{ Such that } \left\{ \begin{array}{l} (MV + PPR) \not\approx 8.0 \\ (MV - NPR) \not\approx 0.0 \\ [(MV - NPR) \wedge (MV + PPR)] \neq 0.0 \end{array} \right.$$

02. **Output:** Entropy(ies) ' H ' found at address(es) $[a, b]$

03. **_begin**

04. $\bar{H}.SetStartOffset(0)$

05. $_ShiftStartingOffset$: /* Goto label for operation mode 2 */

06. $\bar{H}.SetStartOffset(\bar{H}.StartOffset)$

07. $Idx = \bar{H}.StartOffset$ /* Initialize Index to a starting offset */

08. **if** ($\eta == 3$) **then** $\left\{ G_1^{d-1} = \left\lfloor \frac{\bar{H}.Size}{d} \right\rfloor \text{ and } G_d = \bar{H}.Size - \sum_1^{d-1} G \right\}$ such that $2 \leq d \leq \bar{H}.Size$

09. **else** $\{ G_0 = \bar{H}.Size \}$

10. **for** ($Idx; Idx \leq G_r; Idx++$) such that $r = \begin{cases} 0, & \eta == \langle 1|2|4|5 \rangle \\ [1, d], & \eta == 3 \end{cases}$

11. $\bar{H}.SetEndOffset(Idx)$

12. $H = CalculateEntropy(\bar{H})$

13. **if** $\left[\left((H \geq MV) \wedge (H \leq (MV + PPR)) \right) \vee \left((H \geq (MV - NPR)) \wedge (H \leq MV) \right) \right]$

14. **then** $\left\{ \begin{array}{l} \text{Entropy is found at address } [\bar{H}.StartOffset, Idx]; \\ IsEntropyFound = True \end{array} \right\}$

/* If Entropy found and still more data to parse, then advance Idx by 1 and continue */

15. **if** ($Idx < G_r$) **then** $\{ \bar{H}.SetStartOffset(Idx + 1) \}$

/* This is for Operation Mode 2. If no Entropy was found in mode 1, then advance starting offset */

16. **if** $\left[\begin{array}{l} (IsEntropyFound == False) \wedge ((Idx - 1) == G_r) \wedge \\ (H.StartOffset < G_r) \wedge (\eta == 2) \end{array} \right]$

17. **then** $\left\{ \begin{array}{l} \bar{H}.StartOffset++; \\ \text{Goto } _ShiftStartingOffset; (L.05) \end{array} \right\}$

18. **_end**

every single byte in the distribution is different which satisfies the maximum entropy length requirement, the result is a perfect search space which enables the computation of all possible entropy values. This enables us to generate a table containing all possible entropy values which will be fed selectively to Algorithm 2.

For [B.3], if the MV value satisfies the condition presented, then it becomes difficult to compare two floating point values of different accuracies due to compiler and architectural limitations. Thus, if the absolute difference between the entered MV and the calculated entropy value H is less than a given small epsilon value, then the comparison is considered almost equal. However, this condition is not honored in the implementation. It is advised that a value for PPR would be chosen accordingly instead, as this allows greater flexibility when searching for H .

00	01	02	03	04	05	06	07	08	09	0A	0B	0C	0D	0E	0F
10	11	12	13	14	15	16	17	18	19	1A	1B	1C	1D	1E	1F
20	21	22	23	24	25	26	27	28	29	2A	2B	2C	2D	2E	2F
30	31	32	33	34	35	36	37	38	39	3A	3B	3C	3D	3E	3F
40	41	42	43	44	45	46	47	48	49	4A	4B	4C	4D	4E	4F
50	51	52	53	54	55	56	57	58	59	5A	5B	5C	5D	5E	5F
60	61	62	63	64	65	66	67	68	69	6A	6B	6C	6D	6E	6F
70	71	72	73	74	75	76	77	78	79	7A	7B	7C	7D	7E	7F
80	81	82	83	84	85	86	87	88	89	8A	8B	8C	8D	8E	8F
90	91	92	93	94	95	96	97	98	99	9A	9B	9C	9D	9E	9F
A0	A1	A2	A3	A4	A5	A6	A7	A8	A9	AA	AB	AC	AD	AE	AF
B0	B1	B2	B3	B4	B5	B6	B7	B8	B9	BA	BB	BC	BD	BE	BF
C0	C1	C2	C3	C4	C5	C6	C7	C8	C9	CA	CB	CC	CD	CE	CF
D0	D1	D2	D3	D4	D5	D6	D7	D8	D9	DA	DB	DC	DD	DE	DF
E0	E1	E2	E3	E4	E5	E6	E7	E8	E9	EA	EB	EC	ED	EE	EF
F0	F1	F2	F3	F4	F5	F6	F7	F8	F9	FA	FB	FC	FD	FE	FF

Table 1. 1-Gram Byte Hexadecimal Distribution

Figure 1 shows the characterization of entropy with respect to a given number of bytes in a complete holistic heterogeneous search space based on the data in Table 1. As shown, the distribution satisfies the logarithmic equation shown in the *shadow* area. Moreover, the horizontal line, to the left of the graph, shows the minimum number of bytes (in **bold**) that needs to be satisfied in order to get the equivalent entropy value (the complete range of values is shown in Appendix A).

Thus, the question becomes, what is the minimum number of bytes required that satisfy a given entropy value? In Algorithm 2, we address this question ($m = 4$ and $m = 5$), which exploits the characterization presented in Figure 1 in order to reduce the number of steps required when seeking an entropy value. Line [05] shows the minimum number of bytes required for a given entropy value. In the case presented at Lines [06 – 10] ($m = 5$), the value of the step size is relative to the MV or $[MV - NPR]$ sought with respect to the $EntSizeLimit[[MV -$

$NPR]$ array's values at Line [05]. In another words, it advances the index of every round by $EntSizeLimit[[MV + PPR]]$ and changes the starting offset only for the first block that hits $EntSizeLimit[[MV + PPR]]$.

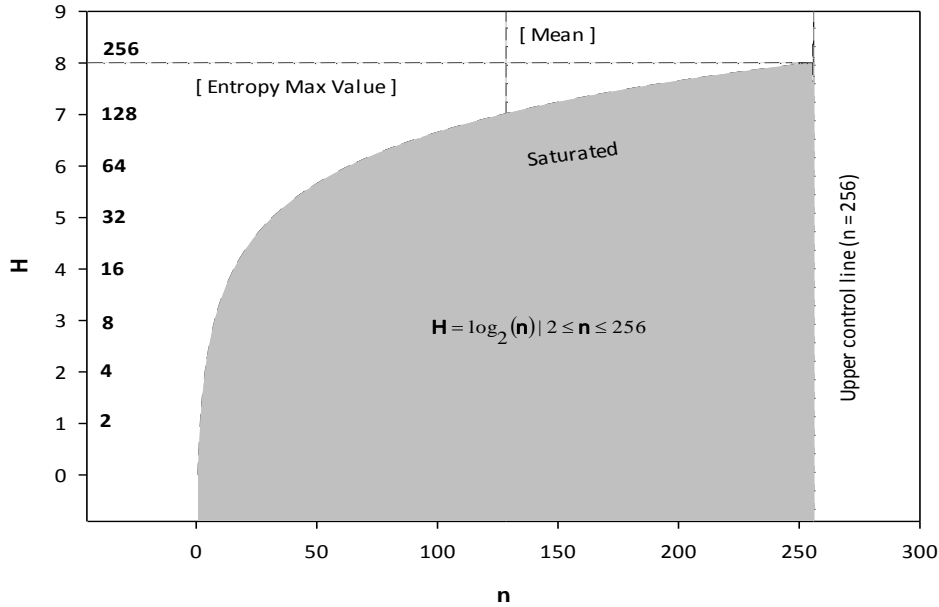


Figure 1. Characterization of Entropy (H) vs Min. Size (n)

Whereas, in the case presented at Lines [11 – 16] ($\eta_j = 4$), the step size is established by taking the *floor* of $(MV - NPR)$, and the block size limit is determined by taking the *ceiling* of $(MV + PPR)$, all with respect to the $EntSizeLimit$ array values. However, this is not intended to be a perfect solution; it is only meant to present a different attack vector (sacrificing accuracy) in order to reduce the processing time required when searching for an entropy value. On the other hand, the characterization array shown could be made more fine-grained by including other sizes as shown in Figure 1 and Appendix A.

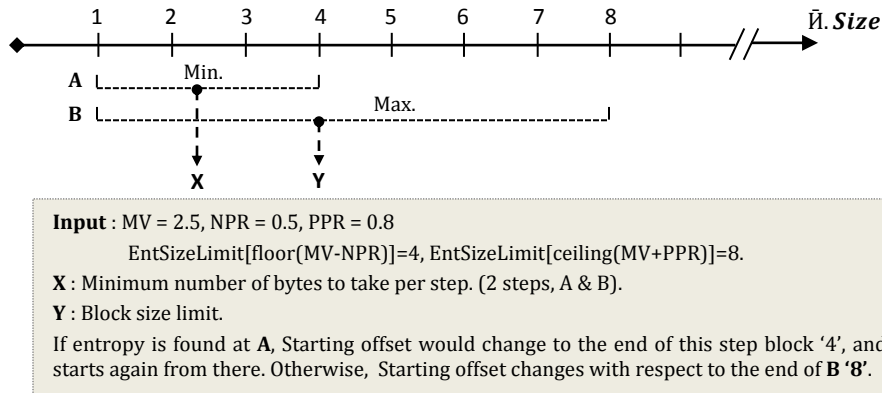


Figure 2. Illustration of Mode 4 ($\eta_j = 4$) operation

Figure 2 shows an example of how Algorithm 2 works in the case presented at Lines [11 – 16] .

Algorithm 2. OPTIMIZED ENTROPY BRUTE FORCER

01. **Input:** Information ' \bar{H} ', Operation Mode ' \bar{m} ', NPR, MV, PPR

With the following definitions: $\bar{m} = 4, 5$

$$\left(\begin{array}{l} (NPR \wedge PPR) \in [0.0, 1.0] \\ MV \in [0.0, 8.0] \end{array} \right) \text{ Such that } \left\{ \begin{array}{l} (MV + PPR) \not\geq 8.0 \\ (MV - NPR) \not\leq 0.0 \\ [(MV - NPR) \wedge (MV + PPR)] \neq 0.0 \end{array} \right.$$

02. **Output:** Entropy(ies) ' \bar{H} ' found at address(es) $[a, b]$

03. **_begin**

04. $\bar{H}.SetStartOffset(0)$

05. $EntSizeLimit[8] = \{2, 4, 8, 16, 32, 64, 128, 256\}$

06. **if** $(MV == X) \mid X \in \mathbb{N} \vee \in \mathbb{Q} \mid 0.0 \leq X \leq 8, NPR = 0 \vee \neq 0, PPR = 0 \vee \neq 0$

07. **if** $(Idx == EntSizeLimit[[MV + PPR]])$

08. $\bar{H}.SetStartOffset(Idx) // Or if entropy found$

09. $\bar{H}.SetEndOffset(Idx)$

10. $Idx += EntSizeLimit[[MV - NPR]]$

11. **While** $(Idx \leq G_r)$

12. **if** $((Idx \% EntSizeLimit[[MV + PPR]] == 0) \wedge NPR \neq 0 \wedge PPR \neq 0)$

13. $\bar{H}.SetStartOffset(Idx) // Or if entropy found$

14. $\bar{H}.SetEndOffset(Idx)$

15. *// What follows is the same as in Algorithm 1.*

16. $Idx += EntSizeLimit[[MV - NPR]]$

17. **_end**

For example, in the scenario presented in Figure 2, the number of steps (worst-case) taken by ($\bar{m} = 4$) (Lines [11 – 16]) is 2 and 12 for ($\bar{m} = 5$) (Lines [06 – 10]). Whereas, in the worst-case scenario for ($\bar{m} = 1$), the number of steps would be 7 and 28 for ($\bar{m} = 2$). Thus, ($\bar{m} = 4$) (Lines [11 – 16]) shows a better performance as compared to the other modes. However, the level of accuracy and computational time vary among all the modes as we will show in the experimentation section. Moreover, other possibilities exist to combine these modes together, for example, mode 1 can be integrated with mode 4.

Note that the order of the modes is different from the ones in the tool. Following is the mapping between the modes presented in the paper ‘P’ and the ones in the final release of the tool ‘T’: ‘P’ (m = 1) -> T (m = 1), ‘P’ (m = 2) -> T (m = 2), ‘P’ (m = 3) -> T (is a separate option invoked via the -b parameter for specifying the block size), ‘P’ (m = 4) -> T (m = 3) and ‘P’ (m = 5) -> T (m = 4).

It is important to note that none of the modes in both algorithms allow overlapping among the found entropies search space. The offsets (starting and ending addresses) of the bytes that constitute the range of entropies found are always in increasing order. And this is a design decision.

III. Computational Complexity Analysis

In this section, we present the computational complexity for the compute intensive modes introduced in section II. The computational complexity for calculating the entropy of a given search space is shown in $E[1]$ (note that B_h represents the size of one byte).

$$\bar{N}.Size + (B_h)^2 \mid B_h = 16 \quad E[1]$$

For mode 1 (worst-case):

$$E[1] \times (\bar{N}.Size - 1) \quad E[2]$$

For mode 2 (worst-case):

$$E[2] + [((\bar{N}.Size - ShiftingOffset) + (B_h)^2) \times ((\bar{N}.Size - 1) - ShiftingOffset)]^2 \quad E[3]$$

$$Such\ that\ 1 \leq ShiftingOffset \leq \bar{N}.Size$$

IV. Experimental Evaluation

In order to provide a comparative analysis on the various modes of operation, 6 binaries (identified as A, B, C, D, E and F) from Windows 7 Professional (x64-bit) with SP 1 were analyzed on an Intel(R) Core(TM)2 Duo CPU P7350 @ 2.00GHz, L2 cache 3072 KBytes, 12-way set associative, 64-byte line size and 4 GBytes of memory (DD3). Moreover, the experiments were conducted using the 32-bit version of Entyzer. For every binary, 8 different entropy values (MV , NPR and PPR) covering almost all the major possible step values are exercised, recording the time it took to find every value as well as the number of entropy(ies) found. Note that the ‘Total’ row values represent the entropy of the whole file. The order of the sought entropy values parameters is (NPR , MV and PPR).

Across all the modes in tables 2, 4, 6 and 8, mode 4 registers the least amount of time it took to search for entropy. However, the degree of accuracy varies dramatically across all the modes. Though the computational time between modes 4 and 5 is close (except F4), the number of entropy(ies) found shows great differences. Thus, a fine-grained approach is almost always better for mining for entropies at the expense of time complexity. The reason why F4 took significantly more time in mode 5 compared to mode 4 is likely to be related to the difference in the number of entropies found. In mode 5, only 12% of the total number of entropies found was detected compared to those in mode 4 (88%).

Therefore, the change in the starting offset was less frequent (which would have reduced the search space at the earliest match) in mode 5.

"A small error in the former will produce an enormous error in the latter"
 - Poincare

		A	B	C	D	E	F
	Size/Bytes	[20480]	[40448]	[51200]	[102400]	[272896]	[651264]
	Total	5.64373	5.88127	5.94664	7.33376	5.34517	6.25283
1	0.3 1.0 0.2	0	0	0	0	1	4
2	0.1 2.0 0.6	0	0	0	0	0	2
3	0.4 3.0 0.6	0	0	0	0	7	3
4	0.4 4.0 0.3	0	0	0	0	94	15
5	0.7 5.0 0.8	0	0	0	0	141	42
6	0.2 6.0 0.1	1	4	5	5	674	623
7	0.2 7.0 0.2	5	20	31	21	674	4950
8	0.8 8.0 0.0	5	19	31	38	674	4950

Table 2. Mode 1 – Time taken to search for entropy

		A	B	C	D	E	F
	Size/Bytes	[20480]	[40448]	[51200]	[102400]	[272896]	[651264]
	Total	5.64373	5.88127	5.94664	7.33376	5.34517	6.25283
1	0.3 1.0 0.2	7524	15596	21003	44845	111813	264521
2	0.1 2.0 0.6	3159	6413	8766	20831	37818	113914
3	0.4 3.0 0.6	1572	3178	4447	11055	17498	57070
4	0.4 4.0 0.3	597	1211	1776	5346	6023	22972
5	0.7 5.0 0.8	259	526	800	3092	2308	10298
6	0.2 6.0 0.1	9	19	27	844	0	740
7	0.2 7.0 0.2	0	0	0	261	0	0
8	0.8 8.0 0.0	0	0	0	107	0	0

Table 3. Mode 1 – Number of entropy(ies) found

		A	B	C	D	E	F
	Size/Bytes	[20480]	[40448]	[51200]	[102400]	[272896]	[651264]
	Total	5.64373	5.88127	5.94664	7.33376	5.34517	6.25283
1	0.3 1.0 0.2	0	0	0	0	1	3
2	0.1 2.0 0.6	0	0	0	0	0	1
3	0.4 3.0 0.6	0	0	0	0	0	0
4	0.4 4.0 0.3	0	0	0	0	0	0
5	0.7 5.0 0.8	0	0	0	0	0	0
6	0.2 6.0 0.1	0	0	0	0	0	0
7	0.2 7.0 0.2	0	0	0	0	0	0
8	0.8 8.0 0.0	0	0	0	0	5	39

Table 4. Mode 4 – Time taken to search for entropy

		A	B	C	D	E	F
	Size/Bytes	[20480]	[40448]	[51200]	[102400]	[272896]	[651264]
	Total	5.64373	5.88127	5.94664	7.33376	5.34517	6.25283
1	0.3 1.0 0.2	4110	8445	11384	23066	59660	137978
2	0.1 2.0 0.6	1726	3464	4630	10781	16109	59222
3	0.4 3.0 0.6	832	1638	2236	5187	7593	28413
4	0.4 4.0 0.3	313	657	908	2358	3083	10991
5	0.7 5.0 0.8	120	268	405	1133	1382	4712
6	0.2 6.0 0.1	0	0	0	160	0	143
7	0.2 7.0 0.2	0	0	0	222	0	152
8	0.8 8.0 0.0	0	0	0	81	0	0

Table 5. Mode 4 – Number of entropy(ies) found

		A	B	C	D	E	F
	Size/Bytes	[20480]	[40448]	[51200]	[102400]	[272896]	[651264]
	Total	5.64373	5.88127	5.94664	7.33376	5.34517	6.25283
1	0.3 1.0 0.2	0	0	0	0	1	4
2	0.1 2.0 0.6	0	0	0	0	0	2
3	0.4 3.0 0.6	0	0	0	0	1	1
4	0.4 4.0 0.3	0	0	1	15	47	497
5	0.7 5.0 0.8	0	0	0	0	8	2
6	0.2 6.0 0.1	0	0	0	4	21	23
7	0.2 7.0 0.2	0	0	0	0	10	77
8	0.8 8.0 0.0	0	0	0	0	5	38

Table 6. Mode 5 – Time taken to search for entropy

		A	B	C	D	E	F
	Size/Bytes	[20480]	[40448]	[51200]	[102400]	[272896]	[651264]
	Total	5.64373	5.88127	5.94664	7.33376	5.34517	6.25283
1	0.3 1.0 0.2	7523	15595	21002	44844	111812	264520
2	0.1 2.0 0.6	2948	6014	8122	20514	36394	108571
3	0.4 3.0 0.6	1380	2822	3943	9845	15650	50539
4	0.4 4.0 0.3	212	522	484	522	332	1330
5	0.7 5.0 0.8	220	445	663	2153	1915	8240
6	0.2 6.0 0.1	9	19	27	1	0	276
7	0.2 7.0 0.2	0	0	0	228	0	0
8	0.8 8.0 0.0	0	0	0	81	0	0

Table 7. Mode 5 – Number of entropy(ies) found

A7 - (0.2 7.0 0.2)			
Blocks	M1	M2	M3
0 - 5120	0	664	0
5120 - 10240	0	675	0
10240 - 15360	0	616	0
15360 - 20480	0	392	0

Table 8. Modes 1, 2 and 3 — Time taken to search for entropy

For mode 2, we took only the sample A7 with one entropy entry, and divided the search space according to mode 3. The results demonstrate the performance impact (time) mode 2 has on brute forcing for entropy. Since no entropy was found (not shown) for modes 1, 2 and 3, all the modes register their worst-case scenarios. The fact that mode 2 didn't reveal any entropy in the search space; it is by definition that none of the other modes would reveal anything.

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APPENDIX A

Size	H										
1	0	51	5.67243	101	6.65821	151	7.2384	201	7.65105	251	7.97154
2	1	52	5.70044	102	6.67243	152	7.24793	202	7.65821	252	7.97728
3	1.58496	53	5.72792	103	6.6865	153	7.25739	203	7.66534	253	7.98299
4	2	54	5.75489	104	6.70044	154	7.26679	204	7.67243	254	7.98868
5	2.32193	55	5.78136	105	6.71425	155	7.27612	205	7.67948	255	7.99435
6	2.58496	56	5.80735	106	6.72792	156	7.2854	206	7.6865	256	8
7	2.80735	57	5.83289	107	6.74147	157	7.29462	207	7.69349		
8	3	58	5.85798	108	6.75489	158	7.30378	208	7.70044		
9	3.16993	59	5.88264	109	6.76818	159	7.31288	209	7.70736		
10	3.32193	60	5.90689	110	6.78136	160	7.32193	210	7.71425		
11	3.45943	61	5.93074	111	6.79442	161	7.33092	211	7.7211		
12	3.58496	62	5.9542	112	6.80735	162	7.33985	212	7.72792		
13	3.70044	63	5.97728	113	6.82018	163	7.34873	213	7.73471		
14	3.80735	64	6	114	6.83289	164	7.35755	214	7.74147		
15	3.90689	65	6.02237	115	6.84549	165	7.36632	215	7.74819		
16	4	66	6.04439	116	6.85798	166	7.37504	216	7.75489		
17	4.08746	67	6.06609	117	6.87036	167	7.3837	217	7.76155		
18	4.16993	68	6.08746	118	6.88264	168	7.39232	218	7.76818		
19	4.24793	69	6.10852	119	6.89482	169	7.40088	219	7.77479		
20	4.32193	70	6.12928	120	6.90689	170	7.40939	220	7.78136		
21	4.39232	71	6.14975	121	6.91886	171	7.41785	221	7.7879		
22	4.45943	72	6.16993	122	6.93074	172	7.42626	222	7.79442		
23	4.52356	73	6.18982	123	6.94251	173	7.43463	223	7.8009		
24	4.58496	74	6.20945	124	6.9542	174	7.44294	224	7.80735		
25	4.64386	75	6.22882	125	6.96578	175	7.45121	225	7.81378		
26	4.70044	76	6.24793	126	6.97728	176	7.45943	226	7.82018		
27	4.75489	77	6.26679	127	6.98868	177	7.46761	227	7.82655		
28	4.80735	78	6.2854	128	7	178	7.47573	228	7.83289		
29	4.85798	79	6.30378	129	7.01123	179	7.48382	229	7.8392		
30	4.90689	80	6.32193	130	7.02237	180	7.49185	230	7.84549		
31	4.9542	81	6.33985	131	7.03342	181	7.49985	231	7.85175		
32	5	82	6.35755	132	7.04439	182	7.50779	232	7.85798		
33	5.04439	83	6.37504	133	7.05528	183	7.5157	233	7.86419		
34	5.08746	84	6.39232	134	7.06609	184	7.52356	234	7.87036		
35	5.12928	85	6.40939	135	7.07682	185	7.53138	235	7.87652		
36	5.16993	86	6.42626	136	7.08746	186	7.53916	236	7.88264		
37	5.20945	87	6.44294	137	7.09803	187	7.54689	237	7.88874		
38	5.24793	88	6.45943	138	7.10852	188	7.55459	238	7.89482		
39	5.2854	89	6.47573	139	7.11894	189	7.56224	239	7.90087		
40	5.32193	90	6.49185	140	7.12928	190	7.56986	240	7.90689		
41	5.35755	91	6.50779	141	7.13955	191	7.57743	241	7.91289		
42	5.39232	92	6.52356	142	7.14975	192	7.58496	242	7.91886		
43	5.42626	93	6.52356	143	7.15987	193	7.59246	243	7.92481		
44	5.45943	94	6.55459	144	7.16993	194	7.59991	244	7.93074		
45	5.49185	95	6.56986	145	7.17991	195	7.60733	245	7.93664		
46	5.52356	96	6.58496	146	7.18982	196	7.61471	246	7.94251		
47	5.55459	97	6.59991	147	7.19967	197	7.62205	247	7.94837		
48	5.58496	98	6.61471	148	7.20945	198	7.62936	248	7.9542		
49	5.61471	99	6.62936	149	7.21917	199	7.63662	249	7.96		
50	5.64386	100	6.64386	150	7.22882	200	7.64386	250	7.96578		